Understanding Alpha Decay

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Abstract

This paper clarifies the relationship between anomaly returns and alpha. The alpha is the expected risk-adjusted excess return on a portfolio that exploits an anomaly. As it is an expected return, the alpha is inversely proportional to the portfolio’s market value. When the alpha decays, for instance after the anomaly has been discovered, the portfolio’s market value increases. Ignoring this effect may cause econometricians to wrongly reject an asset-pricing model. First because, when the alpha decays, the average of past returns leads to an overestimation of the alpha. Further, because asset-pricing tests assume that the alpha has a constant long-term mean. If instead the alpha decays permanently, econometricians may conclude that an anomaly is genuine, when it is in fact extinct. Likewise, when abnormal returns persist after an anomaly’s discovery, the returns may reflect the fact that the anomaly has disappeared, not that it is genuine.

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Over the past thirty years, finance research has uncovered hundreds of strategies for which past returns do not seem to be commensurate with risk (Harvey et al., 2016). These strategies represent anomalies to the extent that their expected risk-adjusted excess returns, or alpha, is positive. Because the alpha is unobserved, empirical studies use averaged risk-adjusted excess returns as a proxy for it. Showing that returns have been abnormally high in the past suffices, in practice, to reject an asset pricing model. This is, however, insufficient to conclude that the returns will continue into the future. Past returns may have been high purely by chance, or because investors were unaware of the anomaly. For this reason, the literature recommends examining returns outside of a study’s original sample. If the returns persist out of sample, then it is worthwhile attempting to understand why this is the case (e.g., Fama (1991)).

In the literature it is assumed that averaged risk-adjusted returns converge to the alpha in small samples. This assumption requires alpha to be stable over time. In this paper I argue that this assumption is questionable. There are many reasons why an anomaly’s alpha may decay. Under rational expectations, alpha decays when the price of risk (or risk itself) decays over time. When there are frictions, the alpha may decay for a variety of other reasons. As time goes by, more investors may become aware of the anomaly; they may have better technology to trade against it, or they may face lower
transaction costs.¹ The possibility of alpha decay obscures the relationship between alpha and returns. This makes returns harder to interpret, both in and out of sample. In sample, asset pricing tests may over-reject the null hypothesis that alpha is zero at the time of testing. Out of sample, risk-adjusted returns may appear statistically significant because the anomaly has disappeared, not because it is genuine.

The core idea in this paper is that alpha and risk-adjusted excess returns are negatively correlated. By definition, the return goes up when alpha decays. This effect is well-known in the context of bonds: when a bond’s yield drops by one percent, the bond price goes up by an amount equal to its duration. The bond duration measures the sensitivity of a bond’s price to a change in interest rates. By analogy, this paper introduces the notion of alpha duration. The alpha duration measures the sensitivity of a portfolio’s market value to a change in alpha.² Multiplying the alpha by the duration gives (approximately) the price change needed to immediately eliminate the anomaly. In other words, the duration turns a flow (the alpha) into a stock.

I present a simple model to show that the alpha duration depends on how persistent an anomaly is. In my model, stock-level alphas decay over time, so that it is necessary to turn over a portfolio to earn a (constant) alpha. The alpha duration depends on how fast stock-level alphas decay. The slower stock-level alphas decay, the larger the duration. Gordon’s growth model arises as a special case with a single stock and permanent alpha. In this case, the duration is as large as the stock’s price-dividend ratio.

How large is the alpha duration, for a typical anomaly? In a recent paper, McLean and Pontiff (2016) examined 97 anomalies and found that returns tend to decay out of sample. Returns decline first at the end of the typical study’s sample, and then further in the period following publication. Inspecting returns around both declines reveals that returns first increase for about two years. This is consistent with alpha decay. A duration of 3, together with a decay speed of about two years, predicts cycles with comparable


²The alpha duration is related, but distinct, from the implied equity duration (Dechow et al., 2004). The implied equity duration measures the sensitivity of a stock’s price to a change in interest rates.
magnitudes.\footnote{McLean and Pontiff attribute the first decrease to statistical biases and the second one to alpha decay (investors eliminating mispricing) after publication. The evidence instead suggests that alpha decay is the primary cause of both decreases. In this connection, Jacobs and Müller (2016) and Lu et al. (2017) argue that anomalies do not decay outside the United States. This conclusion may be premature. If alpha decays with a lag in international data, it may be that these studies measure repricing returns.}

Asset pricing tests ignore the possibility that alpha may decay over the period tested. Consider for instance a prototypical test of the CAPM. Under the null hypothesis, the alpha is zero; it is different from zero (but with a constant long-term mean) under the alternative. The test is well-defined if the alpha has a constant long-term mean. In that case, the average of past excess returns on the zero-beta strategy is an unbiased estimate of the long-term alpha. If alpha decays, but only temporarily, the average return remains unbiased but overestimates alpha in small samples. The issue is more serious if the decay is permanent. In this case, the long-term alpha is no longer constant, so that the average return no longer measures anything meaningful. In both cases, the test over-rejects the null hypothesis that the CAPM holds at the time of testing.

My duration estimate allows me to gauge how much alpha decay creates an upward bias for the average of past excess returns. Chordia et al. (2014) find that anomalies tend to attenuate over time. The returns on a typical anomaly in their sample decline by half in about thirteen years. With a duration of 3, I show that the average of past returns then overestimates the true alpha by approximately 1.4%. This number should be seen as a lower bound that only holds when the decay is temporary. If the decay is permanent, the long-term alpha is no longer constant, and may well have reached zero. In other words, failure to account for alpha decay may explain why researchers have revealed so many anomalies in the recent past.

The remainder of the paper is organized as follows. In Section I, I derive an expression for the alpha duration in a reduced-form model with cross-sectional predictability. In Section II, I discuss the empirical evidence on alpha decay. In Section III, I offer a critique of the ability of standard asset pricing tests to detect anomalies.
I. The Alpha Duration

When the alpha on a portfolio drops by one percent, the portfolio’s market value rises by one percent times the alpha duration. Consider the price of a stock $i$ in a Gordon growth model:

$$P_i = \frac{D_i}{E(r_i) + \alpha_i - g_i}, \quad (1)$$

where $D_i$ is next period’s dividend, $E(r_i)$ is the anomaly-free expected return, and $g_i < E(r_i) + \alpha_i$ is the expected future dividend growth. By assumption, the stock price is anomalous. The stock trades at a discount because in every period its return accrues by an additional alpha, $\alpha_i$, beyond the anomaly-free expected return. When alpha decays by a small amount $d\alpha_i$, the repricing return is

$$\frac{dP_i}{P_i} = -\frac{1}{E(r_i) + \alpha_i - g_i} \times \frac{d\alpha_i}{\text{Duration}}. \quad (2)$$

The return comes with a negative sign: a decline in the alpha requires a positive return. I interpret the sensitivity $1/(E(r_i) + \alpha - g)$ as the alpha duration, by analogy with a bond duration.\(^4\) Observe that the alpha duration equals stock $i$’s price-dividend ratio, $P_i / D_i$. For example, if the price-dividend ratio equals 20, a one percent drop in alpha requires a 20% return. In practice, the alpha duration may be smaller. To show this, in the remainder of this section I review a more general setting where $\alpha_i$ varies randomly over time.

Suppose that the economy now features a continuum of stocks with mass normalized to one. Firm $i$ pays dividends $D_{i,t}$ in each period. Dividends grow on average at a rate $g$, which is common to all firms. In logs, dividend growth equals

$$\Delta d_{i,t+1} = \log \left( \frac{D_{i,t+1}}{D_{i,t}} \right) = g + \varepsilon_{i,t+1}^d, \quad (3)$$

where realized dividend growth shocks $\varepsilon_{i,t+1}^d$ are normal shocks that are identically dis-

\(^4\)Setting $g = 0$, observe that Equation (1) describes the price of a perpetuity.
tributed and potentially cross-correlated. Denote by \( r_{i,t} \) the log return on stock \( i \) that is in excess of the anomaly-free expected return. For simplicity, I normalize this expected return to 0, so that \( r_{i,t} \) describes a risk-adjusted excess return (I describe \( r_{i,t} \) as a return where there is no ambiguity). Define stock-specific alphas as the expected return:

\[
\alpha_{i,t} \equiv E_t(r_{i,t+1}),
\]

where \( E_t(.) \) is the conditional expectation given all information available at time \( t \). Stock-specific alphas now vary over time and can be positive or negative. Specifically, \( \alpha_{i,t} \) follows a zero-mean AR(1):

\[
\alpha_{i,t+1} = \phi \alpha_{i,t} + \varepsilon_{i,t+1}^\alpha,
\] (4)

where \( \varepsilon_{i,t+1}^\alpha \) are identically distributed normal shocks. I assume that alpha shocks are uncorrelated with other alpha shocks and with dividend shocks. Stocks share the same persistence parameter \( \phi \), which is positive and lower than 1. This parameter determines how fast stock-specific alphas converge to zero.

Using Campbell and Shiller (1988)'s log-approximation and the assumptions (3) and (4), log returns can be expressed as

\[
r_{i,t+1} = \alpha_{i,t} + \varepsilon_{i,t+1}^d - \delta \varepsilon_{i,t+1}^\alpha,
\] (5)

where \( \delta \equiv \rho \frac{1}{1 - \rho \phi} \).

The log-linearization constant \( \rho \) is defined by \( \rho = \frac{P/D}{1+P/D} \approx 1 \), where \( P/D \) is the long-term price-dividend ratio. (I leave out the \( i \) index because the ratio is the same for all stocks).

The return \( r_{i,t+1} \) equals the expected return, plus a combination of dividend and alpha shocks. Because alphas are persistent, alpha shocks are multiplied by an additional term \( \delta \). This is the alpha duration: when alpha decays by \( d\varepsilon_{i,t+1}^\alpha = d\alpha_{i,t+1} \), the return, all other things being equal, is

\[
dr_{i,t+1} = -\underbrace{\delta}_{\text{Duration}} \times \underbrace{d\alpha_{i,t+1}}_{\text{Alpha decay}}.
\] (6)

\(^5\)Cochrane (2008), for instance, derives a similar expression for the market return.
Now, consider a portfolio with weights equal to 1 on the stocks for which \( \alpha_{i,t} \) is positive and weights equal to \(-1\) on the remaining stocks for which \( \alpha_{i,t} \) is negative. Denote by \( r_t \) the log excess return on this portfolio. The alpha is the expected return,

\[
\alpha = \int |\alpha_{i,t}| dF(\alpha_{i,t}), \tag{7}
\]

where \( F(\cdot) \) is the c.d.f. of the distribution of the stock-level alphas. This distribution is Gaussian, following the normality assumption for the alpha shocks \( \varepsilon_{i,t}^{\alpha} \). Unlike the alpha on individual stocks, the alpha on the portfolio is positive by construction. In addition, it is constant, following my assumption that alpha shocks are uncorrelated and that there are a large number of stocks.

As in my Gordon growth model, the alpha on the portfolio is constant. One may thus conclude that, as with the Gordon formula, the duration equals the long-term price-dividend ratio. This is not the case. To understand this, suppose that at time \( t + 1 \) all individual alphas change in proportion to their time \( t \) value:

\[
d\varepsilon_{i,t+1}^{\alpha} = d\alpha_{i,t+1} = \theta\alpha_{i,t},
\]

where \( \theta \) measures the speed at which alphas change. Recall that \( \alpha_{i,t} \) may be positive or negative. Because the alpha shock is in proportion to \( \alpha_{i,t} \), all alphas decline in absolute value if \( \theta < 0 \). They all shrink to zero if \( \theta = -1 \). At the portfolio level, the alpha changes by \( \theta \alpha = \int \theta|\alpha_{i,t}|dF(\alpha_{i,t}) \). The repricing return is the sum of all alpha shocks multiplied by the alpha duration:

\[
-\int \delta \times \theta|\alpha_{i,t}|dF(\alpha_{i,t}) = - \underbrace{\delta}_{\text{Duration}} \times \underbrace{\theta \alpha}_{\text{Alpha decay}}. \tag{8}
\]

The alpha duration \( \delta \) that was defined at the stock level therefore also applies at the portfolio level.

It is easier to interpret \( \delta \) by replacing \( \rho \) by its value (Eq. 5):

\[
\delta = \frac{P/D}{1 + (1 - \phi)P/D}. \tag{9}
\]
In my Gordon growth model, alpha is constant, and the duration is equal to the (long-term) price-dividend ratio. This happens here if $\phi = 1$, i.e., if all $\alpha_{i,t}$’s follow random walks. The duration is smaller when $\phi$ is less than one, and is approximately equal to one for $\phi = 0$. In other words, the more persistent the anomaly (the closer $\phi$ is to 1), the larger the duration and thus the bigger the repricing shocks.\footnote{More persistent anomalies also have lower portfolio turnover. Suppose the investor is long on a stock $i$ in $t$. The probability that the investor shorts this stock in $t+1$ is proportional to $F(-\alpha_{i,t}, \alpha_{i,t+1})$, where $F(\cdot, \cdot)$ denotes the cumulative bivariate normal distribution. Though there is no closed-form expression for $F(\cdot, \cdot)$, it is apparent that $F(-\alpha_{i,t}, \alpha_{i,t+1})$ decreases with $\phi$. The same logic applies to stocks she is shorting in $t$. It follows that the slower $\alpha_{i,t}$’s die out (the closer $\phi$ is to one), the smaller the turnover.}

It should be apparent that there cannot be any change in alpha if there is no change in price. This is important when it comes to interpreting the discovery of anomalies. If returns decline after the discovery of an anomaly but one does not observe high returns, it must be the case that the anomaly did not exist to begin with, or that the mispricing wedge was precisely zero at the time of the discovery. This latter case is likely for cyclical anomalies, where alpha may be zero at some point in time (e.g., the discovery of the January effect in June). The next section provides empirical guidance regarding when and how fast markets learn about mispricing around academic publications.

II. Alpha decay around publication

Some of the most prominent anomalies saw their return decay post-publication. For instance, Schwert (2003) remarks that the size factor yielded dismal returns in the twenty years that followed its academic discovery by Banz (1981) and Reinganum (1981). Schwert notes that this discovery coincided with the launch of funds designed to exploit the anomaly. Schwert also finds that the value premium disappeared after 1993. While the value premium was known in the nineties, it is plausible that it received greater attention around the publication of the Fama and French (1993) model. Size and value returns were, in fact, substantial during these periods, which is consistent with alpha decay. Small firms outperformed the market during the two years after 1981, and it is only in
1983 that the size effect presumably vanished. Likewise, high-minus-low returns were high in the years 1992-1993. Returns on individual anomalies are unfortunately noisy, which makes it difficult to establish statistical significance. To overcome this difficulty, researchers have been testing alpha decay on larger samples that pool many anomalies. In this section I review some of this recent evidence.

McLean and Pontiff (2016, hereafter MP) propose an astute design that can be used to analyze alpha decay. The design compares returns in three periods: (i) in sample, (ii) out of sample but before publication, and (iii) after publication. For instance, a new anomaly is published in a journal in 2015 using data up to 2010. MP assume that the anomaly may have been arbitraged away in 2015, but not in 2010. Panel (a) of Figure 1 shows the alpha under three alternatives. First, if the return persists, then the anomaly is probably genuine. Second, the anomaly may be a false discovery. Comparing returns between periods (i) and (ii) reveals a degree of statistical bias due to false discoveries. Third, alpha may decay at publication if investors learn about the anomaly from the published article. Comparing returns between periods (ii) and (iii) should capture this decay. Note that this is true over long periods, so that returns converge to the alpha. Because this condition is unlikely to be satisfied, returns should instead increase when alpha decays.

Panel (b) in Figure 1 reproduces Figure 2 in MP. Black dots represent returns in the years that immediately precede and follow publication, for a typical anomaly. In their sample of 97 long-short portfolios, MP found that the average annual return was 6.9% in sample, it then declined to 4.8% out of sample, and finally to 3.2% post-publication. They conclude that both false discoveries and alpha decay (investors eliminating mispricing) contribute to anomaly attenuations. This conclusion requires returns to cycle at the time of publication, but not at the end of the in-sample period. In contrast, the returns cycle twice. One of the cycles is expected: returns increase after publication, which suggests investors are learning from the published article. The other cycle arises earlier, at the

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7Dimson and Marsh (1999) notice a similar two-year “honeymoon period” after the introduction of an index that trades against the size anomaly in the UK market.
end of the in-sample period. Returns are large in the final year of the in-sample period and again in the first year out of sample.

To interpret these cycles, suppose that alpha decays twice in the data (i.e., there are no false discoveries). The econometrician observes returns and wants to back out the alpha’s trajectory. To this end, I assume that the trajectory exhibits three plateaus during which alpha equals the return. I also assume that alpha decays from one plateau to the next in two years. I then back out the duration and the dates at which alpha decays to match the two cycles. Panel (b) shows the trajectory for alpha (solid blue line), along with the corresponding repricing returns (dashed red line). Figure 1 illustrates the good fit when assuming a duration $\delta = 3$.\(^8\)

How different is this interpretation from MP’s? First and foremost, the presence of return cycles validates their central claim that alpha decays. In particular, one cycle is aligned with the publication year, which confirms that academic research causes alpha to decay. The presence of an earlier cycle at the end of the original sample, however, suggests that alpha decay, not false discoveries, explains why returns decline post sample.\(^9\)

### III. Implications for Asset Pricing Tests

Asset pricing tests ignore the possibility that alpha may decay over time. In the prototypical test considered in the introduction, for instance, alpha is zero under the null hypothesis and non-zero under the alternative. The CAPM was rejected whenever a zero-beta portfolio earned a statistically significant return over the sample period. Rejection of the null hypothesis often convinces researchers to accept an alternative model, that includes the new portfolio as a risk factor. However, what if this significant return reflects the permanent elimination of prior mispricing? It could be that, at the time of testing, the CAPM holds, and yet it is wrongly rejected.

\(^8\)Returns seem to overshoot in the data in years 3-5 post-publication, which can be accounted for by letting alpha move later on.

\(^9\)Chen and Zimmermann (2017) arrive at a similar conclusion by examining the distribution of in-sample t-statistics.
Alpha decay is difficult to measure, especially if it occurs slowly over time. Current evidence only covers several anomalies pooled together. Chordia et al. (2014) in particular found that the returns on twelve anomaly portfolios had been declining over time. Their results are instructive for quantifying the effect of alpha decay on asset pricing tests. In what follows—as in Section II—I take these findings on returns as given, and infer a plausible trajectory for the unobserved alpha. The difference with the previous section is that I now assume that alpha decays over a hypothetical test period.

It is useful to distinguish the current alpha from its long-term value. I index alpha’s current value by time $t$ and denote its long-term value as $\alpha^{LT}$. Alpha decay may be permanent or temporary. When the decay is temporary, the long-term alpha is constant. When the decay is permanent, the long-term alpha varies over time. For instance, there could be a period of slow decay until the long-term alpha reaches zero and stays there.

Figure 2 illustrates Chordia et al. (2014)’s findings under such two cases. As in Figure 1, a solid black line indicates the unobserved alpha, $\alpha_t$, while the solid blue line shows the excess returns on the zero-beta portfolio. In practice, the econometrician observes the average risk-adjusted excess return $\hat{\alpha}^{LT}$, while the quantity of interest is $\alpha^{LT}$. I report $\hat{\alpha}^{LT}$ with a dashed blue line, and the assumed $\alpha^{LT}$ with a dashed black line. Chordia et al. found that $\hat{\alpha}^{LT}$ equals 7.7% per year and that it takes about 13 years for a typical anomaly to attenuate by half. To back out $\alpha_t$ and $\alpha^{LT}$, I let alpha start at some value $\alpha_0$ and decay every period by a constant value $\kappa$ until alpha reaches zero in $T = 2 \times 13$ years. I choose $\alpha_0$ so that $\hat{\alpha}^{LT} = 7.7\%$, using—as in Figure 1—a duration of 3. Finally, black dots illustrate possible trajectories for $\alpha_t$ when $t > T$.

Asset pricing tests make the implicit assumption that, if alpha decays at all, this decay is only temporary. This case, which is shown in Panel (a), supposes that in the future $\alpha_t$ will revert to $\alpha^{LT}$. Because alpha decays over time, the average of past returns overestimates the long-term mean. Equation (8) implies that whenever alpha changes by $\kappa$, the return equals $r_{t+1} = \alpha_t - \delta \kappa$. (Note that I reason that everything else is equal, i.e., I assume dividend shocks average to zero.) The econometrician’s estimate of the
long-term alpha then equals

$$\hat{\alpha}^{LT} = \frac{1}{T} \sum_{t=1}^{T} r_{t+1} = \frac{1}{T} \sum_{t=0}^{T-1} \alpha_t + \frac{1}{T} \sum_{t=1}^{T} -\delta \kappa.$$  \hspace{1cm} (10)

The average is composed of two terms. The first is an average of the true alphas and is, therefore, an unbiased estimate of $\alpha^{LT}$. The second term is the average of the repricing returns. Because the decay is temporary, this term shrinks to zero in long samples. In small samples, however, this term is positive when alpha decays ($\kappa < 0$). In Figure 2, alpha decays by 0.47% per year. With a duration $\delta = 3$, the average of past returns overestimates $\alpha^{LT}$ by 0.47% $\times 3 \approx 1.4\%$, a non-negligible number. Researchers may want to include a 1.4% haircut when testing whether a new portfolio delivers a significant return.

Panel (b) illustrates the case in which the decay is permanent. I now denote $\alpha_t^{LT}$ with a time subscript because the long-term alpha now varies over time. For the sake of illustration, I assume that $\alpha_t = \alpha_t^{LT}$ for all $t$ and that both reach zero at the end of the sample period, $T$. Because $\alpha_t^{LT}$ is no longer constant, the econometrician needs to indicate over which period it is measured. Presumably, she would like to test if $\alpha_T^{LT} \neq 0$. The average of past returns is uninformative in that regard. A positive $\hat{\alpha}^{LT}$ only means that alpha was positive in the past, not necessarily that it is positive at time $T$. The key message here is that, whenever the econometrician suspects that alpha decay may be permanent, she should model this decay. In this example, where alpha decays slowly, the econometrician could test if alpha converges to zero over the sample. If at time $T$ she cannot reject that $\alpha_T^{LT}$ is statistically different from zero, then the asset pricing model is not rejected.

To summarize, asset pricing tests should account for the evidence that anomaly returns have decayed over time. The econometric implications differ according to whether the possible decay is permanent or temporary, but in both cases ignoring alpha decay leads econometricians to overestimate the alpha.
IV. Summary and Implications

Changes in alpha are discount rate shocks, a property that seems to have been overlooked in the literature on cross-sectional return predictability. In this paper I formalize this property in a reduced-form model, in which investors can build up a long-short portfolio to earn a constant alpha. When alpha decays, the return on the portfolio rises in proportion to the alpha duration. More persistent anomalies have larger durations. Using results in McLean and Pontiff (2016), I show that a duration of 3 is a reasonable estimate for a representative anomaly.

It is no news that returns are a poor measure of expected returns. Merton (1980), for instance, notes that very long samples are needed to estimate the equity premium, even when the equity premium is constant. In this article I focus on cross-sectional anomalies. Expected returns on anomalies—alphas—are likely to decay over time, which makes inference even more challenging. Further research could propose asset pricing tests or testing procedures that account for alpha decay. This requires working with expected returns rather than actual returns. Previous literature has explored a variety of approaches to estimating expected returns, which could prove useful for estimating conditional alphas.10 I hope to see work developing this idea in the future.

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10Blanchard (1993), Campbell and Shiller (1988), Pastor et al. (2008), for instance, exploit the accounting relationship between valuation ratios and expected returns. Campbell (1987), Harvey (2001), and many others estimate conditional expected returns by projecting future returns onto conditioning variables. Other approaches include using bond yields (Campello et al., 2008), and survey expectations (Brav et al., 2005) to back out expected stock returns.
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Panel (a) illustrates the research design employed in McLean and Pontiff (2016). Panel (b) compares the returns observed in the data, reproduced from Figure 2 of McLean and Pontiff (2016), together with the assumed alpha and returns. The assumptions on alpha and returns are as follows. Within each subperiod (in-sample, out-of-sample, post-publication) alpha reaches a plateau during which returns and alpha are equal. Alpha decays for two years when transitioning from one plateau to the next. The duration $\delta$ equals 3. Finally, the start of each decline is chosen to match the return patterns in the data.
This figure illustrates the empirical findings in Chordia et al. (2014), that anomaly returns decline by a half in about 13 years. Dashed lines indicate the sample average of returns $\hat{\alpha}^{LT}$ and the long-term mean $\alpha^{LT}$. As in Figure 1, Panel (b), I assume a duration $\delta = 3$. The solid black line shows the assumed alpha, while the solid blue line shows the corresponding return. Alpha decay is temporary in Panel (a) and permanent in Panel (b). In both cases, the assumed trajectory for alpha satisfies $\hat{\alpha}^{LT} = 7.7\%$ (the average anomaly return in Chordia et al. (2014)).