Confirming signals are hard to resist: 
Blessing and curse of information under confirmation bias

Tamara Nunes*  Stefanie Schraeder†

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Abstract

According to empirical evidence individuals pay more attention to confirming than to contradicting evidence. In the context of this confirmation bias, we study the effects of additional information on perception correctness – contrasting the competing effects of total signal precision and the possibility to search for the most suitable signal. We provide the testable hypothesis that managers report bad news in a more diffuse signal compared to good news. This in turn provides an explanation of the dispersion anomaly: dispersed analysts’ earnings forecasts are followed by stock under-performance. Then, we include the confirmation bias in an overlapping generations model with a continuous signal distribution. Several results of more simplified models do not hold true any longer. For example, in our model confirmation bias leads to initial underreaction instead of overreaction. This momentum effect is accompanied by various time varying market participation, announcement day and market depth effects.

JEL Classification: G02, G12

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*University of Lausanne and Swiss Finance Institute, E-mail: tamara.nunes@unil.ch

†University of New South Wales, E-mail: s.schraeder@unsw.edu.au
1 Introduction

Ample empirical evidence documents that individuals react stronger to confirming than to contradicting evidence when updating their beliefs. This effect, called confirmation bias, describes an individual’s search for affirmation, when updating expectations with new information. It is especially pronounced in the case of ambiguous information. Individuals focus on information that is close to their original prior, and interpret noise components as the primary information.

Empirically, the effect of confirmation bias is not only described in the psychology literature, but found in empirical finance articles as well. For example, in the context of stock message boards Park et al. (2013) show that investors preferentially use messages that are in line with their prior perception. Additionally, they find that this effect does not fade with knowledge and experience. Duong et al. (2014) document confirmation bias for glamour stocks.

On the theory side, models like Pouget et al. (2016) and Rabin and Schrag (1999), describe the effects of confirmation bias in stylized models. In these models, exogenous priors, binary signal and state distributions play a key role.\(^1\) Agents tend to one of two extreme states, and ignore contradicting evidence as soon as they develop a preference. Key characteristics of these models are the occurrence of overreaction, bubbles and crashes. However, several predictions, such as the overreaction, are driven by the described simplifying assumptions.

In this paper, we focus on the impact of information on market prices and various market characteristics, in an environment with confirmation biased agents. In a first step, we investigate the impact of information on belief-correctness. While in traditional rational environments, additional information is always increasing the

\(^1\)In a previous version of Pouget et al. (2016), Pouget and Villeneuve (2009) allow for Gaussian signals but consider only the direction of the information with respect to a reference point to identify confirming signals.
perception accuracy, this does not hold true any longer if agents are subject to confirmation bias. Under confirmation bias, the effect of additional information is more ambiguous as the gain in additional information has to be weighed against the chance of having a greater opportunity to choose a convenient signal. This phenomenon sheds new light on the optimal information disclosure policy of managers. Indeed, it leads to the effect that, in the case of negative signals, managers have an incentive to provide information in a more diffuse signal than for positive information in order to reduce the implication of the information on perceived firm value. Consistent with empirical evidence managers have an incentive to reveal positive information earlier in the day than negative information, so that investors can keep being biased towards the initial good information throughout the remaining day. In contrast, in the case of a precise signal confirmation biased investors have no possibility for an alternative interpretation. In this context, Gennotte and Trueman (1996) provide a theoretical rationale for managers to maximize postearnings announcement price by releasing good news during trading hours and bad news when the market is closed. Moreover, this finding is in line with the dispersion anomaly (e.g. Diether et al. (2002) and Avramov et al. (2009)) that analyst dispersion is followed by underperformance.

Second, we include the confirmation bias in equilibrium models. Starting with a representative agent model we find that confirmation bias leads to initial underreaction to new information. Agents are conservative and focus on confirming information. Thus, they concentrate on signals that mitigate the effects of changes in the fundamental value. In an overlapping generations model, especially with endogenous switches between active and passive trading activity, this initial underreaction is mitigated. Due to the resulting mispricing in the case of changes in the fundamental value, new, so far unbiased, investors have an incentive to become
active trader in the market. Becoming confirmation seeking when entering the market, the new signal becomes overrepresented in the entering agents’ expectations. As older agents become passive investors due to an underestimation of the signal’s relevance, we observe an underreaction of smaller magnitude. A consequence is momentum in stock returns especially around earnings announcement days. This post earnings announcement drift, as first documented by Ball and Brown (1968), shows that firms reporting unexpectedly high earnings subsequently outperform firms reporting unexpectedly low earnings. Additionally, Zhang (2006) reports that greater information uncertainty leads to more return anomalies.

Third, the investor population changes over time and especially around announcement days. The market entry due to mispricing gives incentives for new investors to enter. Additionally, due to fading mispricing, older investors suffer from worse performance and as a result leave the market. Thus, after announcement days the investor population becomes younger, less experienced and less biased. Additionally, we observe an enhanced belief dispersion around announcement days, which is in line with the findings of Morse et al. (1991), among others. It results from underreaction and the resulting mispricing triggering new, disagreeing individuals to enter the market. As a result, beliefs among trading individuals differ more after changes in the fundamental. This phenomenon is strongly intensified by the confirmation bias. Indeed, the model demonstrates that confirmation bias increases belief dispersion around announcement days and also on a general level. The more frequent announcements occur, the more disperse the beliefs are. Such an effect is well-known in the context of confirmation bias, essentially in psychology, as the so-called attitude polarization or belief polarization (Lord et al. (1979), Barberis and Thaler (2003) and Taber and Lodge (2006)). The arrival of new information may intensify and not reduce the disagreement between the biased agents, as each
of them looks at news in a way to reinforce their own beliefs.

In addition, several predictions of the models are in line with market stylized facts. The models exhibit volatility clustering (Mandelbrot (1963)), momentum (Jegadeesh and Titman (1993), Hong et al. (2000)), positive correlation between trading volume and volatility (Lamoureux and Lastrapes (1990)) as well as the tendency of biased agents to trade more frequently.

So far, only few papers investigate the confirmation bias in the context of financial markets. Pouget et al. (2016) builds on Rabin and Schrag (1999) and proposes a dynamic asset pricing model in which a part of the traders is subject to the bias. Biased and unbiased agents cohabit. They provide rationale for excess volatility, excess volume, momentum and bubbles in financial markets. However, several of their major results, such as overreaction and bubbles, are primarily driven by the simplifying assumption that both the signals as well as the final states of the world are Bernoulli distributed. Our paper takes a closer look at these results in the context of continuous signal distributions and mature market setup. Park et al. (2013) conduct an empirical investigation on the confirmation bias. They study how stock message boards influence investors trading decisions and investment performance within a field experiment. They show that investors visibly use message boards to seek information that confirms their prior beliefs. Moreover, the confirmation bias tends to make investors overconfident, which affects their investment performance (Park et al. (2013) and Barber and Odean (2001)). Hence, the paper is as well associated to several studies on the effect of overconfidence on financial markets; specifically, to Kyle and Wang (1997), Odean (1998), Benos (1998) and Garcia et al. (2007). Conservatism is another cognitive bias that tends to resemble to the confirmation bias in the sense that agents outweigh prior beliefs but the former refers to the trouble of individuals to update their beliefs upon receiving new information while the latter
corresponds to choosing the information that matches prior beliefs. Barberis et al. (1998) documents underreaction to earnings announcements from agents subject to conservatism. The paper is related to overlapping generations model, such as e.g. Kubler and Schmedders (2015), Gărleanu and Panageas (2014), and Albagli (2015). Compared to these papers our focus is not on the heterogeneity resulting from investment horizon or wealth effects, but on the heterogeneity due to differences in priors when entering the market. Another paper which includes the effects of differences in experience on investment decisions is Schraeder (2016). However, her paper focuses on the effects of availability bias and all signals after a market entrance are weighted equally.

Hence, to the best of our knowledge, this paper is the first to include confirmation bias in an overlapping generations model together with active and passive agents. This allows us to better capture differences between generations and additionally investigate several other empirical model predictions.

The remainder of the paper is structured as follows. Section 2 describes our mathematical representation of the confirmation bias and the effect of the confirmation bias on expectation formation of biased investors. Then, we include the confirmation bias in equilibrium models. We consider a representative agent model in Section 3. Section 4 incorporates the confirmation bias in an overlapping generation model, and finally in an overlapping generation model with endogenous switches of active or passive market activity decisions. Section 5 matches model predictions with empirical evidence. Finally, Section 6 concludes.
2 Confirmation bias and signal processing

2.1 Underlying process and confirmation bias

In this section, we investigate the effect of confirmation bias on belief formation. As confirmation bias describes the impact of priors and past information on the current information processing, we consider a dividend stream with time variation in the changes to the underlying value. We distinguish between non-announcement days on which dividends are normally distributed around the mean, and announcement days (high-volatility days), in which the dividend mean is subject to change. The dividend process is defined by the following set of equations: a measurement and a linear stochastic difference equation

\[ d_t = x_t + \sigma_d \epsilon_{d,t}, \]  
\[ x_t = x_{t-1} + 1_{t \in A} \cdot [\kappa (\mu - x_{t-1}) + \sigma_{x,t}] \cdot \]  

\( \kappa \) captures the mean reversion in the fundamental dividend and \( \sigma_{x,t} \) captures the change in the mean dividend on announcement days \( A \).

At each point in time \( t \), agents receive a number \( n_s \) of independent and normally distributed signals

\[ s_t \sim N \left( x_t, \sigma_s^2 \right). \]

Agents update their believes and filter Equations (1) and (2) to estimate the state of the dividend fundamental part \( x \) using the Kalman filter described in Appendix A.1. In principal, it would be sufficient to focus on the dividend as the only source of information. However, biased agents proceed with the Kalman filter while being

\[ ^2 \text{These signals cover hard and soft information without distinction.} \]
subject to confirmation bias and commit the mistake to focus on the most favorable signal. Indeed, there is empirical evidence that human beings tend to focus on information which is in line with their prior beliefs. Thus, the most favorable and fitting signal is the one which is closest to the a priori perception of the fundamental value for an agent, specified as

$$s_{t}^{m,a} = \arg\min_{i} \left| s_{i,t} - \hat{x}_{t}^{-(c,a)} \right|,$$  \hspace{1cm} (3)

with $\hat{x}_{t}^{-(c,a)}$ the a priori estimate at step t given as a measurement the subjective dividend of a biased agent.

In order to allow for different degrees of confirmation bias, we determine the subjective dividend as $d_{t}^{c,a}$ which is a weighted average of the rational average signal and the most convenient signal previously defined

$$d_{t}^{c,a} = \frac{d_{t} + c s_{t}^{m,a}}{1 + c},$$

where $c$ is the confirmation bias parameter. When $c = 0$, agents correctly update with the dividend process $d_{t}$. For higher values of $c$, the agents are confirmation biased and put weights on the signal, that is closest to their prior beliefs.

### 2.2 The effects of confirmation bias

In an economy, which is only populated by rational agents, additional information is increasing the accuracy of individual beliefs. In the presence of confirmation bias, there are two conflicting aspects of having more signals on perception correctness. On the one hand, more signals increase the accuracy of the mean signal, and there-

\footnote{In a previous version, we consider the case where agents neglect mean reversion. Indeed, agents tend to underestimate mean reversion (e.g. Beshears et al. (2013)). In case of an extreme shock, together with confirmation biased agents, this is associated with underreaction followed by subsequent overreaction.}
fore the correctness of the estimation. On the other hand, having more signals increases the possibility for the agent to choose the best fitting signal from a larger pool. This in turn decreases the need to deal with potential changes in the underlying and decreases the correctness of the estimate. Figure 1 displays the average effect of one additional signal on the perception incorrectness in the fundamental value of the dividend process, computed as $k_t = |\hat{x}_t^{c,a} - x_t|$. In the case of rationality, $c = 0$, perception incorrectness decreases in the number of signals. In the case of strong confirmation bias, $c = 1000$, perception incorrectness increases in the number of signals. For medium degrees of confirmation bias the effect lies between the two extremes.

While the perception incorrectness in the case of rationality is distributed around the true value, in the case of confirmation bias, the deviation goes in the opposite direction of the signal shock. A large number of signals allows biased investors to overweight their preferred one. This leads to the testable hypothesis, that managers may disclose a broad range of information especially around negative announcement. This allows biased investors to underreact to bad news, as they focus on the better news, and favor managers for a time. This is in line with Patell and Wolfson (1982), who find that bad (good) news tend to be released when markets are closed (open). This can be explained by confirmation biased investors, searching for positive information can stick to the good information received in the beginning of the day, even if further bad news is about to come. More recently, Kothari et al. (2009) and Baginski et al. (2015) find that managers continue delaying the disclosure of bad news compare to good news, even after government regulations.

Moreover, we contribute to the dispersion puzzle as described by Diether et al. (2002). If managers are more likely to announce bad information in a diffuse manner, agents underreact to this signal initially and have lower their expectations in
the following times, leading to a lowering in the aggregate market expectations. According to Diether et al. (2002), this effect is stronger for firms that have badly perform over the past year. The opposite holds true for signals that are positive and less diffuse.

3 One generation equilibrium model

We introduce the confirmation bias, as described in Section 2, into an economy with one risky and one riskless asset. The riskless asset is in perfectly elastic supply and generates a constant return $r_f$. The risky asset pays dividends, which follow a process as described in Equations (1) and (2).

In this economy, each agent $a$ maximizes exponential utility of the next period wealth given his perception of the price process

$$\max \mathbb{E}_t^a \left[ U(w_{t+1}^a) \right] = \max \mathbb{E}_t^a \left[ 1 - \exp(-\gamma w_{t+1}^a) \right]. \quad (4)$$

Under the budget constraint, next period wealth equals

$$w_{t+1}^a = w_t^a \cdot (1 + r_f) + y_t^a (p_{t+1} + d_{t+1} - (1 + r_f)p_t). \quad (5)$$

The first order optimality condition results in a demand for the risky asset equal to

$$y_t^a = \frac{\mathbb{E}_t^a [p_{t+1} + d_{t+1}] - (1 + r_f)p_t}{\gamma \text{Var}_t^a (p_{t+1} + d_{t+1})}. \quad (6)$$

Using the market clearing condition ($y_t^a = 1$), we obtain the current market price as a function of next-time period price
\[ p_t^j = \frac{\mathbb{E}_t^a [p_{t+1} + d_{t+1}] - \gamma \text{Var}_t^a (p_{t+1} + d_{t+1})}{1 + r_f}, \quad j \in \mathbb{Z}/n\mathbb{Z}, \] (7)

where \( n \) indicates the frequency at which high-volatility days occur in this economy. We define \( j = \bar{n} = 0 \) as the index corresponding to announcement days. Thus, the index \( j \) describes the number of days passed since the last announcement day. Moreover, we assume mature markets, such that a priori and a posteriori error covariance estimates of the Kalman filter only depend on the position within the cycle of announcement days \( j \), but no longer on \( t \), as the initial uncertainty has vanished.

Following, the guess and verify technique, we conjecture that for all \( j \in \mathbb{Z}/n\mathbb{Z} \), the price process has the following linear structural form

\[ p_t^j = a^j + b^j \hat{x}_t^a. \]

The reader may refer to Appendix A.2 for an analytical derivation of \( a^j \) and \( b^j \).

This model allows us to investigate the effects of confirmation bias on prices, portfolio returns and volatility, as well as the impacts of the number of signals in Section 5. However, the current version the model does not include differences in beliefs due to different starting points; indeed, depending on when agents enter the market, they start their process with different beliefs. Thus, in the next section, we extend the model to account for this issue and consider an overlapping generations equilibrium model. Then, we add, to the overlapping generation model, the possibility for agents to endogenously decide to be active or passive investors in the market.
4 Overlapping generations model

In this extended version of the model, agents enter at different points in time, leading to different priors. We assume that every agent enters the market with rational expectations. The market entrance determines the point in time in which agents start searching for stable perceptions and replace \( \bar{s}_t \) by \( d_t^{c,a} \).

4.1 Fixed lifespan

We consider agents living for \( m \) periods and we label generations according to their point of entry. For example, at time \( t \) generation \( a = t - m \) up to generation \( a = t \) are actively trading in the market as illustrated in Figure 2.

Agents enter the market with unbiased perceptions and start searching for confirmation from then on. This leads to heterogeneity among agents and a different perception of new information. The most convenient signal, as defined in Equation (3), differs among agents, depending on their previous periods believes.

We consider myopic agents, who maximize utility out of next period wealth. Hence, the individual optimization problem follows the Equations (4), (5) and (6). However, compared to the previous section, agents now differ in their perception of the true value of the underlying dividend. Thus, the new market clearing condition aggregates over the different desired portfolio holdings of the trading population

\[ 1 = \frac{1}{m} \sum_{i=1}^{m} y_t^{i-1}. \quad (8) \]

The different generations’ perception regarding the fundamental dividend has been specified above. Regarding the price process, we face two different potential sets of assumptions. Under the first assumption, agents are aware of the functional form of the price process, which in this case is a weighted average of the different
perceptions $d^c_t$. These, however, are not a linear function of past dividends, but a filtering outcome over past signals combined with a biased search for the most favorable signal. Thus, being aware of the functional form of the price process can be considered equivalent to agents knowing about all other agents’ biased believes. The alternative set of assumptions follows the idea that agents perceive themselves as rational and deviations from their perception as noise which fades in the next period. In the following, we consider the second alternative, which leads to individual’s optimal portfolio holdings differing for days non-prior to an announcement and days prior to an announcement, meaning for $j \neq n - 1$ and $j = n - 1$, being respectively equal to

$$y^a_t = \frac{a^{j+1} + (b^{j+1} + 1) \hat{x}^a_t - (1 + r_f)p^j_t}{\gamma \left[ \Sigma_N + (1 + b^{j+1}K^{j+1})^2 \cdot (P_{t+1}^- + \sigma_d^2) \right]} \quad \text{for } j \neq n - 1,$$

$$y^a_t = \frac{a^{j+1} + (b^{j+1} + 1)((1 - \kappa)\hat{x}^a_t + \mu \kappa) - (1 + r_f)p^j_t}{\gamma \left[ \Sigma_N + (1 + b_{j+1}K^{j+1})^2 \cdot (P_{t+1}^- + \sigma_d^2) \right]} \quad \text{for } j = n - 1.$$

In these equations, $\Sigma_N$ captures the impact of noise trading in the price process.\(^4\)

Therefore, we obtain the market realizing price process for days not prior an announcement $j \neq n - 1$,

$$p^j_t = \frac{a^{j+1} + (1 + b^{j+1})x_t^j \sum_{i=1}^m \hat{x}^i_t - \gamma \left[ \Sigma_N + (1 + b^{j+1}K^{j+1})^2 (P_{t+1}^- + \sigma_d^2) \right]}{(1 + r_f)}$$

and for days prior to an announcement $j = n - 1$,

\(^4\)This perceived noise is no white noise, but has a structure. However, as the agents’ perception relative to other market participants changes over time, this structure changes. Therefore, from a subjective point of view the detection of this structure is difficult.
\[
p_i^j = \frac{a^{j+1} + (1 + b^{j+1}) (\kappa \mu + (1 - \kappa) \frac{1}{m} \sum_{i=1}^{m} \hat{x}_t^{t-i}) - \gamma [\Sigma_N + (1 + b^{j+1} K^{j+1})^2 (P_{t+1} - \sigma_d^2)]}{(1 + r_f)}.
\]

\(P_t^r\) is the a priori estimation error covariance of the state estimate. \(K_t\) is the Kalman gain that minimises the a posteriori estimation error covariance \(P_t\) of the state estimate.\(^5\) Parameters \(a\) and \(b\) are defined as in the previous section but differ in the way that they account for the impact of the noise trading.

The realizing price process aggregates all the different perceptions about the fundamental process \(x\). For each generation, the market conditions around their market entrance determines their prior, which significantly influences which information is considered to be in line with their perception. There is a mixed effect of recent information on the realizing price. For those generations, who entered after the information is revealed the information is overly influential, as they ignore mean reversion, and, therefore, overestimate the long-time importance of their information. In contrast agents that have already been in the market around that time, were influenced by confirmation bias when evaluating the different signals transferring the information. Thus, they undervalue the importance of contrary information. Which effect dominates depends on the extent to which a signal is confirming the prevailing average beliefs. Another influencing factor is the relative strength of the cohorts, who consider the signal to be confirmatory. So far, all the generations were equally present in the market. However, the presence of different generations should depend on the profit they can make in the market by entering. This is what we are doing in the following section.

\(^5\)Refer to Appendix A.1 for analytical details on the Kalman procedure and to Appendix A.3 for demand and price functions.
4.2 Endogenous market activity

In this section, we extend the overlapping generations model to an environment in which agents face a market entering decision. The benefits of market participation, resulting from mispricing, increases in the deviation of the average belief in the market (reflected in prices) from the rational expectation, held by entering investors. Entering investors have formed rational prior beliefs about the fundamental value. We assume that they are not (or at least less) affected by cognitive dissonance bias as long as they are not yet involved in the market but once they have entered they are committing the mistake of confirmation bias. Agents are biased as soon as they have entered the market. The rational is that subjective experience in financial market does not tend to insure judgment accuracy as shown in Kahneman and Klein (2009). Figure 3 illustrates the intraperiod time frame of the endogenous market switches from active to passive participation.

If the average market perception deviates from the individual’s perception; agents tend to enter the market. In contrast, if one generation’s perceptions converge towards market perception, market participation converges to zero. Accumulating market entry over time, after one time period, the percentage of agents of one generation having entered actively the market equals at time $t$. We assume entry costs into the market such that a fraction $\eta^a_t$ of the current population belonging to generation $a$ having an incentive to enter

6 Notice that agents do not become more and more biased as they spend time in the market; they are affected by the bias as long as they are active in the market. This is a difference with Schraeder (2016) in which the difference in the individuals’ experience implies heterogeneity among agents.

7 Three specifications are considered. In the first one, an agent remains with the free decision to enter and exit the market but can take both actions only once. The second specification considers that an agent becomes active in the market when she is born and she only stays with the decision of becoming inactive in the market. It can be referred as a free exit decision case. In the third specification, the agents may decide to become again active or inactive even after having become passive or active before.
\[ \eta_t^{l-i} = 1 - \exp(-k \cdot |\hat{p}_t^{l,a} - p_t^{l}|). \]

The variable \( \hat{p}_t^{l,a} \) is the subjective fundamental price \( \hat{p}_t^{l,a} = a^i + b^i \hat{x}_t^a \), and the variable \( p_t^l \) is the market price in an environment with noise trading.

Accumulating market entry of activity over time, after one time period, the percentage of active agents of one generation in the market equals at time \( t \)

\[ \text{active}_{t-i}^l = \text{active}_{t-1-i}^l + (1 - \text{active}_{t-1-i}^l) \cdot \eta_t^{l-i}. \]

We also include the possibility for the agents to become inactive in the market. Once they have become inactive they cannot become active again. The probability, that agents become passive, decreases with the utility they generate out of their investments. Hence, the number of currently active investors in cohort \( a \) at time \( t \) decreases by

\[ \xi_t^{l-i} = (1 - U(w_t^{l-i}))^h, \]

with \( U(w) \) being as defined in Equation (4) and \( h \) a parameter. Thus, the percentage of agents, who after an initial market entry have left the market because of their preferences, equals

\[ \text{passive}_{t-i}^l = \text{passive}_{t-1-i}^l + (\text{active}_{t-1-i}^l - \text{passive}_{t-1-i}^l) \cdot \xi_t^{l-i}. \]

As a result, the percentage of one generation belonging to the active trading population equals

\[ \pi_t^{l-i} = \text{active}_{t-i}^l - \text{passive}_{t-i}^l. \]
The population of newcomers is high in times of higher disagreement and deviation from the fundamental, which is the case if there have been recent shocks. This is also the time in which older agents leave the market due to poor prior performance. For a more detailed description of the participation dynamics can be found in Section 5.2.

The time-varying active trading population has an impact on the market clearing mechanism, because only the active trading population directly influences prices through trade. Thus, not only changes in portfolio holdings itself, but also market entry and exit impact prices. The market clearing condition adapts to

\[
1 = \frac{\sum_{i=1}^{m} \pi_i^{t-i} \cdot y_{t-i}}{\sum_{i=1}^{m} \pi_i^{t-i}}. \tag{9}
\]

For all other days except days prior to an announcement, \((j \neq n - 1)\), the pricing equation equals

\[
p_j^t = \frac{a^{j+1} + (1 + b^{j+1}) \left( \frac{1}{\Pi_t} \sum_{i=1}^{m} \pi_i^{t-i} \hat{x}_t^{t-i} \right) - \gamma \left[ \Sigma_N + (1 + b^{j+1}K^{j+1})^2(P_{t+1}^- \sigma_d^2) \right]}{(1 + r_f)}. \]

For days prior to announcement days \((j = n - 1)\) the pricing equation equals

\[
p_j^t = \frac{\kappa \mu + (1 - \kappa) \frac{1}{\Pi_t} \sum_{i=1}^{m} \pi_i^{t-i} \hat{x}_t^{t-i} - \gamma \left[ \Sigma_N + (1 + b^{j+1}K^{j+1})^2(P_{t+1}^- \sigma_d^2) \right]}{(1 + r_f)}. \]

In these equations, \(\Pi_t = \sum_{i=1}^{m} \pi_i^{t-i}\) equals the total market participation. Like-
wise, $P_t^-$ is the \textit{a priori} estimation error covariance of the state estimate and $K_t$ is the Kalman gain. Parameters $a$ and $b$ are defined as previously.

## 5 Effects of confirmation bias

In this section, we give market predictions that follow from the different model specifications presented above.

### 5.1 Underreaction

Figure 4 compares a sample price path under confirmation bias with the rational one. It shows that in a model with a continuous state distribution together with mature markets, the confirmation bias results in an underreaction to new information. This differs from the results derived in Rabin and Schrag (1999) and Pouget et al. (2016), as their finding rely on the assumption of a binary distribution with two extreme signals and outcomes in young markets. In a continuous state distribution, agents choose to pay attention especially to those realizations, which are closest to their original prior. In the case of an extreme event, agent pays most attention to signals close to their prior beliefs, which is one of the least extreme signals. Hence, she underreacts as illustrated in Figure 4.

In the multiple generations model, another effect interferes with the initial underreaction. After an extreme signal, new generations enter the market without being affected by previous priors. At first, they do not underreact, but form rational beliefs about the underlying fundamental value. Due to the underreaction prevailing in the market they observe deviations of the market price from the fundamental value. This potential for future profits results in an incentive for market entry. As a first consequence, this market entry mitigates the impact of biased agents on prices.
Second, the newly entering agents start with a prior, which is strongly influenced by the extreme signal. Underreacting to future information, a strong cohort of newly entering agents leads the information remaining in the market for an extended period of time. Figure 5 highlights the difference between the price process when agents are biased against rational ones in the case of a exogenous market entry and exit. Contrary to the previous representative agent model, agents, once they have just entered the market, behave rationally and mix with other generations that are biased.

Then, we consider the model with endogenous switches and Figure 6 shows the average deviation of an average active investor’s belief from the fundamental value, in the case of a negative shock to the fundamental. We again first observe the underreaction, namely a deviation in the opposite direction of the shock.

Chan et al. (1996) report momentum in the short run while DeBondt and Thaler (1985) show that returns are negatively correlated at long horizon. In addition, Hong and Stein (1999) propose an asset pricing model to provide a unified account of first underreaction together with subsequent overreaction.

5.2 Age generation and market participation

In the model with endogenous market activity, around an extreme negative shock, the incumbent agents underreact to new information due to confirmation bias. This leads to an underperformance in the following time. As agents’ willingness to actively invest in the market depends on their performance, the underperformance leads to a larger fraction of active investors becoming passive. As a result, the percentage of older agents decreases after a shock to the fundamental on high volatility days. In contrast, the newcomer generations are not influenced by previous priors. Prices deviate from the fundamental value. This creates an incentive for more
newcomers, unbiased agents to become active. The mechanism is demonstrated in Figure 7. In Panel (b) of Figure 7, there is an extreme negative shock, the agents, born before such an event, strongly suffer from their prior beliefs and from under-performance. These generations demonstrate a clear break in their trading activity compared to agents that are born at the event or after and tend to trade more.

Moreover, Figure 8 points to the same conclusion by considering the effect of the negative shock on the one-period utility function of various generations. It is clear that agents that have entered formerly to the shock suffer a decrease in their utility compared to post-shock born agents.

In addition, Figure 9 gives the average age of the agents participating in the market when agents are fully rational, slightly subject to the confirmation bias and highly biases. When there is some confirmation bias, the average age of the active agents is inferior to the average age of rational agents only, regardless of announcement days. Biased agents tend to survive less time in the market.

5.3 Trading volume and its relation to volatility

In our model, especially in the endogenous switches, trading volume consists of two components. The first component results from agents changing their portfolio holdings, while remaining in the market. The second component results from agents buying and selling their shares, when becoming active or passive investors.

The first component of trading volume $PV$ is defined as the change in an individual’s portfolio holdings, relative to his previous periods holdings

$$PV_t = \sum_{i=1}^{m} \pi_{t-i}^t |y_{t-i}^t - y_{t-1}^t|.$$ 

The second component of trading volume $EH$ is defined as the change in holdings
of agents who change their trading status

\[ EH_t = \sum_{i=1}^{m} \left( \left| y_{t-i} - \frac{1}{m} \cdot \delta_{\text{active}}^i \right| + \left| y_{t-1} - \frac{1}{m} \cdot \delta_{\text{passive}}^i \right| \right), \]

with \( \delta_{\text{active}} \) and \( \delta_{\text{passive}} \) being defined as the number of agents starting and stopping to trade actively.

Total trading volume then is defined as the sum of these two elements such that simply \( TV_t = PV_t + EH_t \).

A higher degree of confirmation bias leads to a higher trading volume, both due to changes in the holdings of active investors, as well as due to market entrance and exit. A higher degree of confirmation bias changes the disagreement in the investor population and results in trade due to portfolio restructuring. The higher confirmation bias coefficients also lead to a higher entry or exit trading volume. This can be attributed to the effect, that not only the disagreement among active investors increases with confirmation bias, but also the disagreement of still passive agents with the aggregate market belief. This, in turn, triggers market entry. Moreover, the differences in belief also lead to a wider distribution in portfolio returns and a higher frequency of market exit due to negative performance shocks. This finding is empirically supported among others by Park et al. (2013), who show that biased investors have the tendency to trade more frequently.

As especially extreme events trigger both disagreement as well as changes in the market prices, we observe a positive correlation between trading volume and volatility. Table II shows that the correlation coefficient is increasing in the confirmation bias parameter. This is consistent with numerous empirical papers on trading volume and volatility interaction, such as Jones et al. (1994).

In Table III, we report the serial correlation in returns for different strength of the confirmation bias and the number of signals. Positive serial correlation is stronger
as agents are more biased and as the number of signals increases. It verifies the fact that when agents are not fully rational and many signals are available to them, agents are even more seeking for information confirming their prior beliefs, leading to the existence of momentum. A testable hypothesis is that the more disperse is the information disclosed by managers, the stronger is the momentum in returns. A close hypothesis has been examined in Hong et al. (2000) in which they show that momentum comes from gradual information flow, especially negative information and that there is more momentum in stocks for which information gets out more slowly. In this case, the dispersion of information is due to the time that takes information to flow.

Table IV illustrates the effect of confirmation bias on volatility clustering. The higher the number of signals, and the stronger the effect of confirmation bias on belief formation, the stronger is also the effect of volatility clustering. Around announcement days investors with a higher degree of confirmation bias tend to search for a confirming signal and underreact. Thus, the process of adapting to the new situation takes longer and clustering is stronger.

5.4 Market depth

In our model, we measure market depth as the inverse derivative of prices with respect to the demand

$$\lambda = \left( \sum_{i=1}^{m} \frac{\partial p}{\partial y_{t-1}} \right)^{-1}.$$ 

To estimate this price impact numerically, we use a simple centered finite difference approximation. In our estimation, we focus on the overlapping generation models in the case of a constant market population and in the case of endogenous market participation. Table V shows the results.
Two opposite effects of confirmation bias on liquidity come to play: total market participation and the willingness to trade. If market participation is constant, market depth is increasing in the confirmation bias, in absolute terms. In the case of endogenous participation, market depth increases as well in confirmation bias (i.e. the price impact of demand is smaller, in absolute terms). Although, we demonstrate that the overall market participation is decreasing in the confirmation bias level (see Figure 9), willingness to trade appears to have a stronger effect and the effect is stronger when we allow agents to actively participate or not in the market, as the two-sample t-test shows.

For comparison purposes, in the context of overconfidence, the papers by Odean (1998), Benos (1998) and Garcia et al. (2007)\(^8\) examine, amongst others, the impact of overconfidence on market depth. Odean (1998) and Benos (1998) show that the presence of overconfident traders increase market depth. In contrast, more recently Garcia et al. (2007) demonstrate that this does not hold anymore when information is endogenously acquired. Market depth is decreasing in the overconfidence level, as rational agents tend to leave the market. This is in line with our findings in the case of endogenous participation. Furthermore, as in all pre-mentioned papers regarding the impact of overconfidence, in our model a higher degree of confirmation bias leads to higher volume.

### 5.5 Announcement day effects

Announcement days differ from low-volatility days in several ways. Around announcement days the effect of confirmation bias on prices is most pronounced. Prices deviate from their fundamental value especially around these times. As time passes

\(^8\)Overconfidence is often described as an overestimation by traders of the precision of their private or public signals. Again, in this paper, confirmation bias leads agents to choose signals that are closer to their prior beliefs and to neglect mean-reversion.
by and agents have to face the fact that the signals constantly deviate from their perception and biased agents exit due to underperformance, the effect of confirmation bias fades. Indeed, Figure 10 shows an increase in the dispersion of beliefs around announcement days and at a general level in the presence of the confirmation bias. Moreover, concerning the frequency of news, we observe that in the presence of the confirmation bias, the dispersion of beliefs tends to be higher when announcement days are less spaced in time, creating an increase in the general level of the dispersion of beliefs. This phenomenon favors the so-called attitude polarization, such an effect is known to stem from confirmation bias. Indeed, as it is too difficult for these agents to change their beliefs; new information tends to be distorted to fit prior beliefs and this in turn widens disagreements even more among the agents. We observe that, the more frequent the announcements arrive, the more disperse beliefs are. Disagreements among agents may augment as new announcements arrive often. This leads to an empirical and testable prediction on the effect of the frequency of information disclosed by managers on beliefs dispersion of analysts. In this line, Wilde (2005) investigates for Swiss firms the impact of financial reporting frequency on stock market. She shows that companies with more regular interim reports do not tend to face less uncertainty among analysts, i.e. the standard deviation among analyst forecasts is not decreasing with the reporting frequency.

Finally, we look at the effect of announcement days on market depth, trading volume and market volatility for different values of the confirmation bias. Table VI reports the absolute value of the average market depth, average trading volume and volatility for days prior to announcement days and on announcement days. As expected, market volatility is higher on announcement days, as these days are high volatility days. Trading volume follows an increasing trend as announcement days approach. In this sense, Chordia et al. (2001) demonstrate that, in the case of U.S.
equities, trading volume increase prior to gross domestic product and unemployment announcements, but it falls back to its normal level on announcement days. Such results are attributed in Chordia et al. (2001) to the dispersion in beliefs of market agents’ prior to macroeconomic announcements and the entry in the market of more informed traders. Chae (2005) reports similar results as Chordia et al. (2001) but only before unscheduled announcements. In case of scheduled announcements, agents avoid trading as they perceive high information asymmetry. This pattern for the trading volume holds for strictly positive confirmation bias parameter and stands from more change in holding a period before announcements.

6 Conclusion

This paper investigates the effects of information in the context of confirmation bias in an overlapping generations model, with endogenous participation. The confirmation bias is a source of heterogeneity among agents, which is independent from endowment effects and trading horizon. In the context of this more complex model we are able to provide additional intuition for the effects of confirmation bias. In contrast to already existing models on confirmation bias, we show that confirmation bias is leading to initial underreaction.

Other model implications are a higher trading volume of biased agents. We find a positive relation between trading volume and volatility, a time variation in the investor’s trading experience, as well as announcement day and liquidity effects of the confirmation bias on financial markets. Moreover, we provide a testable hypothesis regarding managers’ optimal information policy towards investors. In the case of bad news, a higher dispersion in information results in a reduction in market reaction. If transferring good news, however, a strong market reaction is
desirable and as a result less dispersed signals should be observed. Together with
the underreaction effect of confirmation bias, this effect is in line with the dispersion
anomaly.

Moreover, the effect that an initial misreaction finally results in a subsequent
over-correction in the opposite direction, may not only be linked to confirmation
bias and can be subject to future research also in other fields.
A Appendix

A.1 Kalman filtering

The optimal filtering technique, that permits to estimate the state $x$ from Equations (1) and (2), differs between announcement days and non-announcement days. In the following, we first define as $\hat{x}_{t}^{-}$ to be the \textit{a priori} estimate at step $t$, given the process prior to $t$. $\hat{x}_{t}$ is the \textit{a posteriori} state estimate at step $t$, given measurement $d_t$. Then, we discuss the filter time update equations.

The \textit{a priori} and \textit{a posteriori} estimate error covariance are respectively

$$P_{t}^{-} = \mathbb{E}\left((x_t - \hat{x}_{t}^{-})(x_t - \hat{x}_{t}^{-})^T\right), \text{ and}$$

$$P_{t} = \mathbb{E}\left((x_t - \hat{x}_{t})(x_t - \hat{x}_{t})^T\right).$$

For non-announcement days, we obtain

$$\hat{x}_{t}^{-} = \hat{x}_{t-1},$$

$$P_{t}^{-} = P_{t-1}.$$  

For announcement days, the filter time update equations equal

$$\hat{x}_{t}^{-} = (1 - \kappa)\hat{x}_{t-1} + \kappa \mu,$$

$$P_{t}^{-} = (1 - \kappa)^2 P_{t-1} + \sigma_x^2.$$

The filter measurement update equations are equivalent for both announcement and non-announcement days.
\[ K_t = P_t^{-1} (P_t^{-1} + \sigma_d^2)^{-1}, \]

\[ \hat{x}_t = \hat{x}_t^{-} + K_t \cdot (d_t - \hat{x}_t^{-}), \]

and

\[ P_t = (1 - K_t)P_t^{-}. \]

\( K_t \) is the Kalman gain that minimizes the a posteriori estimation error covariance \( P_t \).

### A.2 Derivation of pricing equation parameters: One generation

In this section, we briefly state the derivations for the parameters of the pricing equation in a representative agent model. Under the assumption that we are dealing with mature markets, we have \( K_t = K^j \) with \( j = t \mod n \). As \( j \in \mathbb{Z}/n\mathbb{Z} \), we also obtain that \( j + 1 = j - n + 1 \). The first and second conditional moments of the price and the dividend processes differ between announcement days and non-announcement days and are equal to

\[ \mathbb{E}_t^a [d_{t+1}] = E_t^a \left[ x_{t+1} \right] = \hat{x}_t^a \quad \text{for } j \neq n - 1 \]

\[ \mathbb{E}_t^a [d_{t+1}] = E_t^a \left[ x_{t+1} \right] = (1 - \kappa)\hat{x}_t^a + \kappa \mu \quad \text{for } j = n - 1 \]

\[ \mathbb{E}_t^a [p_{t+1}] = a^{j+1} + b^{j+1} \hat{x}_t^a \quad \text{for } j \neq n - 1 \]

\[ \mathbb{E}_t^a [p_{t+1}] = a^{j+1} + b^{j+1} [(1 - \kappa)\hat{x}_t^a + \kappa \mu] \quad \text{for } j = n - 1 \]

\[ \text{Var}_t^a (d_{t+1}) = P_{t+1}^{-} + \sigma_d^2 \]

\[ \text{Var}_t^a (p_{t+1} + d_{t+1}) = (1 + b^{j+1}K^{j+1})^2 \cdot (P_{t+1}^{-} + \sigma_d^2). \]
Inserting these results into the pricing equation, we obtain that
for \( j \neq n - 1 \)

\[ p_t^j(1 + r_f) = a^{j+1} + (b^{j+1} + 1)\hat{x}_t^a - \gamma [(1 + b^{j+1}K^{j+1})^2 \cdot (P_{t+1}^- + \sigma_d^2)] \]

and for \( j = n - 1 \)

\[ p_t^j(1 + r_f) = a^{j+1} + (b^{j+1} + 1)((1 - \kappa)\hat{x}_t^a + \kappa \mu) - \gamma [(1 + b^{j+1}K^{j+1})^2 \cdot (P_{t+1}^- + \sigma_d^2)]. \]

Separating the term with and without \( \hat{x}_t^a \), together with an iterative procedure, we get the parameters \( a \) and \( b \) for any \( n \).

**A.3 Derivation of pricing equation parameters: Multiple overlapping generations**

Derivations are close to the ones in Section A.2 except that, here, we account for noise trading. In an overlapping generations model and in mature markets, the first and second conditional moments of the price and the dividend processes are as follows

\[
\begin{align*}
\mathbb{E}_t^a [d_{t+1}] &= E_t^a [x_{t+1}] = \hat{x}_t^a \quad \text{for } j \neq n - 1 \\
\mathbb{E}_t^a [d_{t+1}] &= E_t^a [x_{t+1}] = (1 - \kappa)\hat{x}_t^a + \kappa \mu \quad \text{for } j = n - 1 \\
\mathbb{E}_t^a [p_{t+1}] &= a^{j+1} + b^{j+1}\hat{x}_t^a \quad \text{for } j \neq n - 1 \\
\mathbb{E}_t^a [p_{t+1}] &= a^{j+1} + b^{j+1}[(1 - \kappa)\hat{x}_t^a + \kappa \mu] \quad \text{for } j = n - 1 \\
\text{Var}_t^a (d_{t+1}) &= P_{t+1}^- + \sigma_d^2
\end{align*}
\]
\[
\text{Var}_t^n (p_{t+1} + d_{t+1}) = (1 + b^{j+1}K^{j+1})^2 \cdot (P_{t+1}^- + \sigma_d^2) + \Sigma_N.
\]

The remainder of the derivations are similar as in Section A.2.
References


Figure 1: Perception correctness as a function of the number of signals. This figure shows the perception correctness of agents as a function of the number of signals. Subfigure (a) illustrates the case of $c = 1000$, which is the case of highly confirmation biased agents. As a comparison, Subfigure (b) shows the rational case of $c = 0$. Perception incorrectness decreases in the number of signals for rational agents. For highly confirmation biased agents in contrast the effect of signal selection is dominant. In this case signal incorrectness increases in the number of signals.

Figure 2: Overlapping generation. At each point in time one generation enter the market and lives for $m$ periods. As a result, there are always $m$ different generations actively trading in the market.
Figure 3: **Intraperiod time frame of market events.** First dividends are paid out. Thus, time $t$ dividends are paid to investors who bought the stock at time $t-1$. Then, agents change their trading status, before investors start trading. Thus, the periods of market entrance is the first period of trading. Market exit happens one period after the last active trade.

![Figure 3](image)

Figure 4: **Price processes: in a representative agent model.** This figure plots the average price process for rational (green) and biased agents (blue) in the case of a representative agent model. At time $t = 10$ there is a negative shock which is three standard deviations below the mean. In the case of confirmation bias, the price process underreacts to new information and completely lacks behind. The price process is computed using $c = 1000$ for the confirmation biased price process, $n = 4$, $n_s = 30$, $\mu = 1.5$, $\sigma_d = 0.5$, $\sigma_x = 0.5$, $\kappa = 0.2$, $r_f = 0.01$, and $\gamma = 0.01$. 

![Figure 4](image)
Figure 5: Price processes: in an overlapping generations model. This figure plots the average price process for rational (green) and biased agents (blue) in the case of exogenous market entry and exit in Subfigure (a) and the difference between the two processes in Subfigure (b). At time $t = 40$, there is a negative shock which is three standard deviations below the mean. We first observe the underreaction, namely a deviation in the opposite direction of the shock, and a deviation in the same direction of the shock. The price process is computed using $c = 1000$ for the confirmation biased price process, $n = 4$, $m = 40$, $n_s = 100$, $\mu = 1.5$, $\sigma_d = 0.25$, $\sigma_x = 0.5$, $\kappa = 0.2$, $r_f = 0.01$, and $\gamma = 0.01$. 
Figure 6: Belief deviation. This figure shows the average deviation of an average active investor’s belief from the fundamental value, in the case of a negative shock to the fundamental. The results are calculated in a model with endogenous market switches. We first observe the underreaction, namely a deviation in the opposite direction of the shock, and then a deviation in the same direction of the shock. The average price reaction is calculated from $N = 1'000$ price paths and parameter values equal to $c = 1000$, $n = 4$, $n_s = 100$, $m = 40$, $\mu = 1.5$, $\sigma_d = 0.25$, $\sigma_x = 0.5$, $\kappa = 0.2$, $r_f = 0.01$ and $\gamma = 0.01$. 

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Figure 7: Life-cycle participation for generations around a negative shock. This figure shows the life-cycle participation for seven generations around a negative shock. The negative shock hits on the vertical blue dashed line. The other curves represent the life-cycle of seven generations; in blue there is a generation born 5 time period before the shock, in black 3 time period before, in green only 1 before. The cyan generation is born at the negative shock time period while the generations in magenta is born 1 time period after, the yellow 3 and finally the red generation 5 time period after. In Subfigure (a) illustrates the case of no extreme negative shock. As a comparison, Subfigure (b) shows the effect on participation when an extreme negative shock happens. We choose parameters values equal to $c = 1000$, $n = 4$, $n_s = 100$, $m = 40$, $\mu = 1.5$, $\sigma_d = 0.25$, $\sigma_x = 0.5$, $\kappa = 0.2$, $r_f = 0.01$ and $\gamma = 0.01$. Life-cycles are calculated over 1’000 iterations.
Figure 8: One-period utility around a negative shock. This figure plots the utilities for four generations. In black, we plot the generation that was born 3 time period before the event. In green, the generation that was born 1 time period prior the shock while the cyan and the magenta curves correspond to generations that were respectively born at the shock time and 1 period after. We choose parameters values equal to $c = 1000$, $n = 4$, $n_s = 100$, $m = 40$, $\mu = 1.5$, $\sigma_d = 0.25$, $\sigma_x = 0.5$, $\kappa = 0.2$, $\tau_f = 0.01$ and $\gamma = 0.0105$. One-period utility are calculated over 1'000 iterations.
Figure 9: Average Age. This figure plots the average age throughout an announcement-cycle in the case of a negative shock to the fundamental. After announcement days the average age decreases as old agents leave the market due to bad performance and new agents enter, as they disagree with the average (biased) perception reflected in prices. We choose parameters values equal to $c = 1000$, $n = 4$, $n_s = 100$, $m = 40$, $\mu = 1.5$, $\sigma_d = 0.25$, $\sigma_x = 0.5$, $\kappa = 0.2$, $r_f = 0.01$ and $\gamma = 0.01$. 
**Figure 10: Belief dispersion.** This figure plots the variance of beliefs of the fundamental value. Announcement days happen every $n$. Moreover, a negative shock of two standard-deviation happens at the same time of an announcement. The variance results from $N = 1'000$ price paths and parameter values equal to $m = 16$, $n_a = 100$, $\mu = 4$, $\sigma_d = 0.25$, $\sigma_x = 0.5$, $\kappa = 0.2$, $r_f = 0.01$ and $\gamma = 0.01$. 
Table I: Different sources of trading volume. This table reports the different sources of trading volume. It splits the total trading volume $TV$ into the trading volume due to changes in the portfolio holdings of active market participants $PV$ and the trading volume generated by market changing between active and passive trading behaviour $EH$. Both sources of trading volume increase through confirmation bias. The parameters used in the simulation are $n = 4, n_s = 100, m = 16, \mu = 1.5, \sigma_d = 0.25, \sigma_x = 0.5, \kappa = 0.1, r_f = 0.01$ and $\gamma = 0.01$.

<table>
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<tr>
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<th>$c = 1000$</th>
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<tr>
<td>PV</td>
<td>0.0</td>
<td>3.1</td>
<td>38.3</td>
</tr>
<tr>
<td>EH</td>
<td>0.2</td>
<td>0.8</td>
<td>7.4</td>
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<tr>
<td>TV</td>
<td>0.2</td>
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</table>

Table II: Correlation between trading volume and volatility. The table reports pairwise linear correlation coefficient between trading volume and squared returns. We observe the correlation coefficient to be positive. It is increasing in the confirmation bias parameter. Parameters equal $n = 4, n_s = 100, m = 40, \mu = 1.5, \sigma_d = 0.25, \sigma_x = 0.5, \kappa = 0.1, r_f = 0.01$ and $\gamma = 0.01$. ** indicates that we reject the null hypothesis of zero correlation between rational and realized returns at the 1% significance level.

<table>
<thead>
<tr>
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<tr>
<td>$\rho$</td>
<td>-0.00</td>
<td>0.16***</td>
<td>0.24***</td>
</tr>
</tbody>
</table>

Table III: Serial correlation in returns. The table reports serial correlation in returns for different numbers of signals up to four lags and different confirmation bias parameter. The remaining model parameters are $n = 4, \mu = 4, \sigma_d = 0.5, \sigma_x = 0.5, \kappa = 0.1, r_f = 0.05$ and $\gamma = 0.075$.

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<tr>
<td>lag 1</td>
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<td>lag 2</td>
<td>0.018</td>
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<td>lag 3</td>
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Table IV: Serial correlation in squared returns. The table reports serial correlation in volatility for different numbers of signals up to four lags and different confirmation bias parameters. The remaining model parameters are $n = 4$, $\mu = 4$, $\sigma_d = 0.5$, $\sigma_x = 0.5$, $\kappa = 0.1$, $r_f = 0.05$ and $\gamma = 0.075$.

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<td>$c = 0$</td>
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</tr>
<tr>
<td>lag 1</td>
<td>0.004</td>
<td>0.013</td>
</tr>
<tr>
<td>lag 2</td>
<td>-0.035</td>
<td>-0.029</td>
</tr>
<tr>
<td>lag 3</td>
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<td>0.000</td>
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Table V: Absolute average value of market depth approximation. The table reports the average value of market depth approximated in the absence ($\hat{\lambda}_{\text{constant}}$) and in presence of endogenous market participation ($\hat{\lambda}_{\text{endogenous}}$). Results are in absolute terms. Market depth is increasing with confirmation bias in the multiple overlapping generations model, while it is decreasing when one allows for endogenous participation. Two-sample t-tests are performed and we do reject the null of equal means for $c = 0$ and $c = 1$, and for $c = 0$ and $c = 1000$, as well as for $c = 1$ and $c = 1000$ at level 5% in both models. ** indicates that the null hypothesis that $\hat{\lambda}_{\text{constant}}$ and $\hat{\lambda}_{\text{endogenous}}$ are equal in means can be rejected at the 5% level. Parameters equal $n = 4$, $n_s = 100$, $m = 16$, $\mu = 1.5$, $\sigma_d = 0.25$, $\sigma_x = 0.5$, $\kappa = 0.1$, $r_f = 0.01$ and $\gamma = 0.01$ and run 1’000 simulation paths.

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<td>0.24</td>
<td>1.91**</td>
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<tr>
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<td>0.03</td>
<td>0.55*</td>
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</table>
Table VI: Relative to announcement days, average market depth, volume and volatility. The table reports absolute value of the average market depth, in the case of endogenous market participation. Parameters equal $n = 4$, $n_s = 100$, $m = 16$, $\mu = 1.5$, $\sigma_d = 0.25$, $\sigma_x = 0.5$, $\kappa = 0.1$, $r_f = 0.01$ and $\gamma = 0.01$ and run 1’000 simulation paths.

\[
\begin{array}{c c c c}
\text{Market depth} & c = 0 & c = 1 & c = 1000 \\
\hline
n - 3 & 0.00 & 0.28 & 1.42 \\
n - 2 & 0.00 & 0.41 & 3.42 \\
n - 1 & 0.00 & 0.20 & 1.00 \\
n & 0.00 & 0.10 & 1.96 \\
\end{array}
\]

\[
\begin{array}{c c c c}
\text{Trading volume} & c = 0 & c = 1 & c = 1000 \\
\hline
n - 3 & 0.16 & 2.31 & 27.45 \\
n - 2 & 0.16 & 2.90 & 33.16 \\
n - 1 & 0.16 & 7.98 & 83.85 \\
n & 0.16 & 2.36 & 40.53 \\
\end{array}
\]

\[
\begin{array}{c c c c}
\text{Volatility} \\
\hline
n - 3 & 0.0002 & 0.0003 & 0.0003 \\
n - 2 & 0.0002 & 0.0002 & 0.0002 \\
n - 1 & 0.0001 & 0.0002 & 0.0002 \\
n & 0.0051 & 0.0027 & 0.0017 \\
\end{array}
\]